Received: *May 12, 1998*, Accepted: *June 22. 1998*

D-branes in curved spacetime: the Nappi–Witten background

Sonia Stanciu

Theoretical Physics Group, Blackett Laboratory, Imperial College
Prince Consort Road, London SW7 2BZ, UK
E-mail: s.stanciu@ic.ac.uk

Arkady Tseytlin

Theoretical Physics Group, Blackett Laboratory, Imperial College
Prince Consort Road, London SW7 2BZ, UK
Institute of Theoretical Physics
University of California
Santa Barbara, CA 93106, USA
E-mail: tseytlin@ic.ac.uk

Abstract: We find exact D-brane configurations in the Nappi–Witten background using the boundary state approach and describe how they are related by T-duality transformations. We also show that the classical boundary conditions of the associated sigma model correspond to a field dependent automorphism relating the chiral currents and discuss the correspondence between the boundary state approach and the sigma model approach.

Keywords: Sigma Models, D-branes, Conformal Field Models in String Theory.

JHEP06(1998)010

Contents

1. Introduction

The *boundary state* formalism (see, e.g., [1, 2, 3, 4]) has become in the last years one of the main approaches to the study of D-branes [5] in various type II string backgrounds [6, 7, 8, 9, 10, 11]. In particular, this approach showed that D-branes probe a new aspect of the background geometry, namely the geometry of submanifolds.

However, despite its success in unravelling some of the structure underlying D-brane geometry, the geometric information obtained through the boundary state approach is often rather difficult to interpret in terms of the more standard sigma model approach. For instance, in the Calabi-Yau case the geometric picture of D-branes wrapping around supersymmetric cycles emerges only in the large-volume approximation in which the metric is taken to be essentially flat. In the case of group or coset spaces this connection seems even more difficult to attain, as the fields in terms of which the conformal structure is realised do not seem to have an obvious geometric (that is, spacetime) interpretation.

In order to make progress in the understanding of D-branes in curved spaces one needs to somehow bridge the gap between the boundary state approach and the corresponding sigma model interpretation. In this paper, we attempt to do this, by choosing a simple string background [12] and studying the corresponding D-brane configurations from both points of view.

In order to apply the boundary state approach one has to start with an exact string solution whose 2d CFT description is explicitly known. On the other hand, in order to give D-branes a spacetime interpretation we need to consider a string background which has a sigma model realisation. One of the few known classes of exact string backgrounds (see, e.g., [13]) with spacetime interpretation is represented by WZW models [14]. In this case spacetime is a group manifold and the exact (non-perturbative) conformal invariance is guaranteed by the Sugawara construction.[1]

We have chosen the Nappi-Witten (NW) solution [12] for the two reasons. From the point of view of the boundary state approach, the simplicity of the corresponding WZW model allows us to solve exactly for the boundary conditions and hence find explicitly the D-brane configurations. On the other hand, despite its simplicity, this model describes a curved homogeneous four-dimensional spacetime with Minkowski signature. This distinguishes our analysis from previous studies in which D-branes in compact spaces were considered. Here the D-branes *worldvolumes* will be fully embedded in the NW spacetime. For the sake of simplicity we consider mainly the bosonic case (generalisation to supersymmetric case is straightforward).

The paper is organised as follows. We start, in Section 2, by reviewing the NW background and the underlying CFT. In Section 3 we describe the boundary state approach to finding D-brane configurations in a closed-string background. We write down the appropriate boundary conditions and solve them explicitly, obtaining two classes of solutions, each of them described by a matrix depending on three real parameters. Further, in Section 4, we study the geometry of these solutions, and find that one of the two classes describes D3- and D1-branes, whereas the other one describes euclidean D0-branes. In Section 5 we consider T-duality transformations in the space of boundary states, and explain how they map between various D-branes.

In Section 6 we re-interpret these D-brane configurations as static classical solutions of the Born-Infeld action for a D-brane probe in the NW background.

[1] As in the previous studies, we ignore back reaction of D-branes on spacetime, i.e. investigate possible D-branes that can be embedded in a fixed geometry. In some cases (representing BPS superpositions of several branes) one knows how the supergravity solution changes once one adds a new D-brane. In (most of) these cases, however, one does not have a 2d CFT description of the corresponding string solutions and thus of the corresponding D-branes.

We then turn, in Section 7, to the sigma model approach. We start with a detailed analysis of the boundary conditions of general 2d sigma models and show that the standard boundary conditions in terms of fields can be recast in terms of the chiral currents. This is particularly important for the WZW models, where the chiral currents are conserved and generate, upon quantisation, the affine algebra which underlies the conformal invariance of the model. We then specialise to NW background and conclude that these classical boundary conditions obtained from the sigma model action are described by a matrix which is closely related to the one obtained in Section 3 using the boundary state approach, with the essential property that the parameters are actually field-dependent functions.

Some details of the space-time embedding of the D-string solution are given in Appendix A. In Appendix B we briefly discuss the supersymmetric generalisation of the construction of the boundary states in the Nappi-Witten model.

2. The Nappi–Witten background

The Nappi-Witten background [12] (for generalisations of this model see for instance [15, 16, 17]) is an exact four-dimensional (super)string solution defined by the following WZW action

$$I[g] = \int_\Sigma \langle g^{-1}\partial g, g^{-1}\bar{\partial} g\rangle + \tfrac{1}{6}\int_B \langle g^{-1}dg, [g^{-1}dg, g^{-1}dg]\rangle\,, \tag{2.1}$$

where the fields g are maps from a closed orientable Riemann surface Σ to the Lie group $\mathbf{G}$, which is to be thought of as the simply-connected group corresponding to the $d = 4$ Lie algebra $\mathfrak{g}$ with the generators $X_a = (P_1,\ P_2,\ J,\ K)$ satisfying

$$[P_1, P_2] = K\,, \qquad [J, P_1] = P_2\,, \qquad [J, P_2] = -P_1\,. \tag{2.2}$$

This algebra is a central extension of the two-dimensional Poincaré algebra. Its important feature is that it possesses an invariant metric $\langle X_a, X_b\rangle = G_{ab}$ given by

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & b & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\,,$$

where b is a real parameter (which can be, in principle, absorbed in a redefinition of the generators).

The group manifold $\mathbf{G}$ can be parametrised as follows

$$g = e^{a_i P_i} e^{uJ+vK}\,, \tag{2.3}$$

where (a_1, a_2, u, v) play the rôle of the spacetime coordinates in string theory. In terms of these fields (1) becomes a sigma-model action, with the spacetime metric and 2-form given by

$$ds^2 = da_i da_i - \epsilon_{ij} a_i da_j du + b du^2 + 2du dv\,, \tag{2.4}$$

$$B = \tfrac{1}{2}\epsilon_{ij} a_i da_j du\,. \tag{2.5}$$

This background describes a gravitational plane wave in $d = 4$ space-time with signature $(+ + + -)$.

The exact conformal invariance of this model is based, as is well known, on its infinite-dimensional symmetry group $\mathbf{G}(z) \times \mathbf{G}(\bar{z})$ characterised by the conserved currents

$$J(z) = -\partial g g^{-1}\,, \qquad \bar{J}(\bar{z}) = g^{-1}\bar{\partial} g\,. \tag{2.6}$$

These currents generate an affine Lie algebra $\widehat{\mathfrak{g}}$ described by

$$J_a(z) J_b(w) = \frac{G_{ab}}{(z-w)^2} + \frac{f_{ab}{}^c J_c(w)}{z-w} + \mathrm{reg}\,, \tag{2.7}$$

in the holomorphic sector, and similar OPEs in the antiholomorphic sector. Furthermore, the generalisation of the Sugawara construction to the nonsemisimple algebra case [12, 18, 19, 20] gives us a CFT with the central charge equal to four and the energy-momentum tensor

$$\mathsf{T}(z) = \Omega^{ab}(J_a J_b)(z)\,,$$

where Ω^{ab} is the inverse of the invariant metric, $\Omega_{ab} = 2G_{ab} + \kappa_{ab}$, and κ is the Killing form of $\mathfrak{g}$:[2]

$$\Omega = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2b-2 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}.$$

3. Boundary states

The approach of constructing boundary states of closed string theories (see, e.g., [1]) is based on the requirement of conformal invariance. In open string theories one has to impose constraints on the boundary conditions such that the conformal symmetry is not broken. Then the boundary can be thought of as a closed string state where the left– and right–moving conformal structures are related in a *consistent* way. The strategy is basically to try to find the consistent boundary conditions on the fields, preserving the "overall" conformal structure.

[2]It is not a priori clear that G and Ω can simultaneously be nondegenerate. However, this is true for self-dual Lie algebras [21], and our $\mathfrak{g}$ falls in this class.

JHEP06(1998)010

In the case of a background described by a WZW model [22, 8, 9] the natural variables, that is, the fields in terms of which the conformal structure of the model is realised, are the affine currents. Hence it is on them that one imposes the boundary conditions

$$J_a(z) + R^b{}_a \bar{J}_b(\bar{z}) = 0\,. \qquad (3.1)$$

These boundary conditions have to satisfy the following consistency requirements:

(i) They preserve conformal invariance, that is

$$\mathsf{T}(z) - \bar{\mathsf{T}}(\bar{z}) = 0\,,$$

at the boundary.[3]

(ii) They preserve the infinite-dimensional symmetry of the current algebra (2.7). One can alternatively argue that this condition is imposed by the fact that the boundary condition for the energy-momentum tensor is not restrictive enough to determine the allowed configurations uniquely.

In order to analyse these conditions, it is convenient to define the map $R : \mathfrak{g} \to \mathfrak{g}$, defined as $R(X_a) = X_b R^b{}_a$, where $X_a \in \{P_1, P_2, J, K\}$. Then the first requirement translates into

$$R^T \Omega R = \Omega\,, \qquad (3.2)$$

whereas the second one imposes on R the two conditions, corresponding to the first and second order pole of the OPE in (2.7), respectively:

$$[R(X_a), R(X_b)] = R([X_a, X_b])\,, \qquad R^T G R = G\,. \qquad (3.3)$$

Notice that (3.2) and the second condition in (3.3) are equivalent in the particular case of our Lie algebra $\mathfrak{g}$, because of the special form of the metric and of the Killing form. We are therefore left with only two conditions, stating that R must be an *automorphism* of $\mathfrak{g}$ which *preserves the metric.*

One of the appeals of the NW model is that it is simple enough to allow us to solve for these conditions explicitly. We obtain two families of solutions: the first

$$R_I = \begin{pmatrix} \cos\phi & -\sin\phi & r_1 & 0 \\ \sin\phi & \cos\phi & r_2 & 0 \\ 0 & 0 & 1 & 0 \\ -r_1\cos\phi - r_2\sin\phi & r_1\sin\phi - r_2\cos\phi & -\frac{1}{2}(r_1^2 + r_2^2) & 1 \end{pmatrix} \qquad (3.4)$$

[3] More generally, the holomorphic and the antiholomorphic sectors are related by an automorphism of the corresponding CFT. However, since the automorphism group of the Virasoro algebra is trivial, we have the above condition.

and the second

$$R_{II} = \begin{pmatrix} \sin\phi & \cos\phi & r_1 & 0 \\ \cos\phi & -\sin\phi & r_2 & 0 \\ 0 & 0 & -1 & 0 \\ r_1\sin\phi + r_2\cos\phi & r_1\cos\phi - r_2\sin\phi & \frac{1}{2}(r_1^2+r_2^2) & -1 \end{pmatrix} \tag{3.5}$$

Both solutions depend on three real parameters, r_1,r_2 and ϕ. In fact, the "moduli space" in each case is $\mathbb{R}^2 \times S^1$. As follows from (8), $(\det R)^2 = 1$. Indeed, we find that $\det R_I = 1$, $\det R_{II} = -1$. This suggests that R_I (R_{II}) will describe odd (even) dimensional D-branes.

One can represent R_I in a more compact form as

$$R_I = Ad(e^{\phi J} e^{\epsilon_{ij} r_i P_j})\,,$$

which shows that R_I is given by the inner automorphisms of $\mathfrak{g}$. R_{II} can be related to R_I by noticing that

$$R_I(\phi, r_1, r_2) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} R_{II}(\phi, r_2, r_1)\,.$$

4. D-brane solutions and geometry

So far we have determined the boundary states of the Nappi–Witten model which preserve conformal invariance and the algebraic structure underlying it. However, it is not clear that any such boundary state will have a *geometric interpretation* as a D-brane. First, we have to clarify the way in which the geometric information defining the D-brane worldvolume is encoded in our boundary conditions. In other words, we have to define what we mean by Neumann and Dirichlet boundary conditions. This step is particularly important in our case, since the boundary conditions are not imposed directly on the fields but rather on the *chiral currents*.

The best way of deciding which boundary condition is a Neumann one and which is a Dirichlet one is to evaluate J and $\bar{J}$ at the origin (or in the flat space limit). We find that at $a_1 = a_2 = u = v = 0$

$$\begin{aligned} J|_0 &= -\partial a_1 P_1 - \partial a_2 P_2 - \partial u J - \partial v K\,, \\ \bar{J}|_0 &= \bar{\partial} a_1 P_1 + \bar{\partial} a_2 P_2 + \bar{\partial} u J + \bar{\partial} v K\,. \end{aligned}$$

This shows that, for the currents, the Neumann boundary conditions for the fields correspond to $J_a = -\bar{J}_a$, whereas the Dirichlet one – to $J_a = \bar{J}_a$.

To analyse the boundary states we have obtained and identify the D-branes they describe we have to study the eigenvalues and eigenvectors of the linear operator R. We shall consider the two classes of solutions separately.

4.1. R_I boundary states

The eigenvalues of R_I are given by

$$\lambda_{1,2} = e^{\pm i\phi}\,, \qquad \lambda_3 = \lambda_4 = 1\,.$$

In fact, R_I can be brought, by a real change of basis, to the following "standard" form

$$R_I^{st} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\,,$$

which makes it easy to identify the D-brane configurations R_I describes.

For $\phi \neq \pi$ our solution describes a *D3-brane* with "generalised" Neumann (or "mixed") boundary conditions along the first two directions (for $\phi = 0$, R_I becomes simply the identity matrix, which is equivalent to having Neumann boundary conditions in all 4 directions). One may be tempted to conclude that we have a background gauge field at the boundary (cf. [5, 23]) whose field strength depends on ϕ. However, since we are dealing with a WZW model, a non-trivial 2-form field B_{ab} may account for the above boundary conditions. We shall come back to this point in Section 6 where we will analyse the boundary conditions from the point of view of the corresponding sigma model.

In the case of $\phi = \pi$ the eigenvalues of R_I read

$$\lambda_1 = \lambda_2 = -1\,, \qquad \lambda_3 = \lambda_4 = 1\,,$$

so that this boundary state describes a *D1-brane* (D-string). The tangent space of its worldsheet is spanned by the eigenvectors of R_I corresponding to the $+1$ eigenvalues:

$$Y_3 = \tfrac{1}{2} r_1 P_1 + \tfrac{1}{2} r_2 P_2 + J\,, \qquad Y_4 = K\,.$$

On the other hand, the eigenvectors corresponding to the Dirichlet directions read

$$Y_1 = P_1 - \tfrac{1}{2} r_1 K\,, \qquad Y_2 = P_2 - \tfrac{1}{2} r_2 K\,.$$

One can now check that the tangent vectors to the Neumann directions form an abelian subalgebra, which we will write formally as $[N, N] \subset N$. This means that the corresponding worldsheet is actually a submanifold of the target manifold. Moreover the tangent vectors to the Dirichlet directions satisfy $[D, D] \subset N$, as expected.

Notice that in this case $R_I^2 = \mathbf{1}$, which implies that the metric splits with respect to the directions tangent and normal to the worldsheet of the D-string

$$\Omega = \Omega_N + \Omega_D\,,$$

where Ω_N denotes the induced metric on the tangent space to the D-brane and Ω_D is the metric induced in the normal bundle.

We can go further and obtain a spacetime description of this D-string solution. The generic tangent vector to the D-string worldsheet is of the form $\alpha Y_3 + \beta Y_4$, with α and β arbitrary parameters. This generates a surface which is nothing but the worldsheet of the D-string. On the other hand, every point on this surface can be parametrised as a group element (2.3). This yields an equation

$$e^{\alpha Y_3 + \beta Y_4} = e^{a_i(\alpha,\beta) P_i} e^{u(\alpha,\beta) J + v(\alpha,\beta) K} ,$$

determining the spacetime fields a_i, u, v in terms of the parameters α, β, i.e. the surface describing the D-string worldsheet,

$$a_1(\alpha,\beta) = \tfrac{1}{2} r_1 \sin\alpha + \tfrac{1}{2} r_2 (\cos\alpha - 1) , \tag{4.1}$$
$$a_2(\alpha,\beta) = -\tfrac{1}{2} r_1 (\cos\alpha - 1) + \tfrac{1}{2} r_2 \sin\alpha , \tag{4.2}$$
$$u(\alpha,\beta) = \alpha , \tag{4.3}$$
$$v(\alpha,\beta) = \beta + \tfrac{1}{8}(r_1^2 + r_2^2)(\alpha - \sin\alpha) . \tag{4.4}$$

Using the expression for the spacetime metric (2.4) one can now compute the induced metric on the D-string worldsheet:

$$ds^2 = \left[b + \tfrac{1}{4}(r_1^2 + r_2^2) \right] d\alpha^2 + 2 d\alpha d\beta . \tag{4.5}$$

Notice that in the particular case of $r_1 = r_2 = 0$ we have

$$a_1(\alpha,\beta) = a_2(\alpha,\beta) = 0 , \quad u(\alpha,\beta) = \alpha , \quad v(\alpha,\beta) = \beta , \tag{4.6}$$

which means that the worldsheet of the corresponding D-string coincides with the (u,v) plane.

The metric (4.5) describes a two-dimensional Minkowski spacetime, and therefore the worldsheet of the D-string is flat in the induced metric. Moreover it is easy to see that its extrinsic curvature also vanishes. This is because the worldsheet is actually the manifold of a Lie subgroup **H** of **G** and relative to a bi-invariant metric on **G**, any Lie subgroup **H** is totally geodesic. This is equivalent to the vanishing of the second fundamental form and therefore of the extrinsic curvature.

4.2. R_{II} boundary states

R_{II} case can be analysed along similar lines. The eigenvalues turn out to be completely independent of the parameters,

$$\lambda_1 = 1 , \qquad \lambda_2 = \lambda_3 = \lambda_4 = -1 .$$

The Neumann eigenvector always exists and is given by

$$Y_1 = \begin{cases} (1+\sin\phi) P_1 + \cos\phi P_2 + \frac{1}{2}[r_1(1+\sin\phi) + r_2\cos\phi] K & \text{for } \phi \neq \frac{3\pi}{2}; \\ P_2 + \frac{1}{2} r_2 K & \text{otherwise.} \end{cases} \tag{4.7}$$

As can be easily checked, it has positive norm. This means that at any point on the integral curve of this vector field, the tangent space to the spacetime has an orthogonal decomposition into a 1-dimensional subspace tangent to the curve and a 3-dimensional subspace normal to it. The subspace tangent to the curve is the eigenspace of R_{II} with eigenvalue $+1$. However, a generic R_{II} will not act like $-\mathbf{1}$ on the subspace normal to the curve, i.e., it will not give rise to the Dirichlet boundary conditions. For this to be the case we will be forced to restrict the values of r_1, r_2, and ϕ.

Indeed, R_{II} has always one Dirichlet eigenvector Y_2 given by K. The existence of the other two Dirichlet eigenvectors requires the diagonalisability of R_{II}. This in turn requires that the parameters obey the following relation:

$$r_1\left(\cos\tfrac{\phi}{2}-\sin\tfrac{\phi}{2}\right)=r_2\left(\cos\tfrac{\phi}{2}+\sin\tfrac{\phi}{2}\right)\ . \tag{4.8}$$

In this case we can conclude that these boundary states describe a *D0-brane* whose tangent space is spanned by Y_1. Since Y_1 has positive norm, we deduce that it is a euclidean D0-brane. We can distinguish four cases; in each of these cases one can write down the corresponding eigenvectors. Y_1 is given by (4.7) and $Y_2 = K$. The other two Dirichlet eigenvectors Y_3 and Y_4 are given in the following table.

(ϕ, r_1, r_2)	Y_3	Y_4
$\phi\neq\frac{\pi}{2},\frac{3\pi}{2},\ r_1r_2\neq 0$	$P_1-\frac{2r_1}{r_1^2+r_2^2}J$	$P_2-\frac{2r_2}{r_1^2+r_2^2}J$
$\phi\neq\frac{\pi}{2},\frac{3\pi}{2},\ r_1=r_2=0$	$P_1-\frac{\cos\phi}{1-\sin\phi}J$	J
$\phi=\frac{\pi}{2},\ r_2=0$	P_2	$J-\frac{1}{2}r_1P_1$
$\phi=\frac{3\pi}{2},\ r_1=0$	P_1	$J-\frac{1}{2}r_2P_2$

In this case one can also check that $[N,N]\subset N$ (trivially, since N is one-dimensional) and that $[D,D]\subset N$. Moreover, in each of the cases listed above, $R_{II}^2=\mathbf{1}$ and the metric splits as

$$\Omega=\Omega_N+\Omega_D\,.$$

The spacetime description of this D0-brane configuration can be determined by parametrising the world-line of the brane by α, such that

$$e^{\alpha Y_1}=e^{a_i(\alpha)P_i}\,e^{u(\alpha)J+v(\alpha)K}\ ,$$

which allows us to write down the explicit expressions for the curve. For $\phi\neq\frac{3\pi}{2}$ we find

$$a_1(\alpha)=(1+\sin\phi)\alpha \tag{4.9}$$
$$a_2(\alpha)=\cos\phi\ \alpha \tag{4.10}$$
$$u(\alpha)=0 \tag{4.11}$$
$$v(\alpha)=\tfrac{1}{2}\left[r_1(1+\sin\phi)+r_2\cos\phi\right]\alpha\,. \tag{4.12}$$

The induced metric on the world line is given by $ds^2 = 2(1+\sin\phi)d\alpha^2$. For $\phi = \frac{3\pi}{2}$ the world line of the D0-brane is described by

$$a_1(\alpha) = 0\,, \quad a_2(\alpha) = \alpha\,, \quad u(\alpha) = 0\,, \quad v(\alpha) = \tfrac{1}{2} r_2 \alpha\,. \tag{4.13}$$

and the corresponding expression for its induced metric is $ds^2 = d\alpha^2$.

5. T-duality

In this section we shall study the effect of abelian T-duality transformations on the D-brane configurations we have found above. The structure of the duality group here is slightly different from the one in the case of flat backgrounds. In particular, the WZW model is *self-dual* under an abelian T-duality transformation [24, 25, 26].

By a T-duality transformation we will understand a map [8, 9], at the level of the fields, which preserves the conformal structure of the model. More precisely, we will consider a map that acts trivially on the antiholomorphic sector of the theory and preserves the Virasoro algebra of the holomorphic sector.[4] Hence if the original theory is described by the currents $J_a(z)$, $\bar{J}_a(\bar{z})$, the dual theory will be described by

$$J'_a = T^b{}_a J_b\,, \qquad \bar{J}'_a = \bar{J}_a\,,$$

where the map $T : \mathfrak{g} \to \mathfrak{g}$ is defined by $T(X_a) = T^b{}_a X_b$.

An arbitrary T-duality transformation is defined by the properties:

(i) It preserves the conformal structure of the model. Since the antiholomorphic CFT will be trivially preserved, this means that the T-duality map at the level of the currents will have to preserve the Sugawara energy-momentum tensor of the holomorphic sector

$$\mathsf{T}(z) = \mathsf{T}'(z)\,.$$

(ii) It preserves the infinite-dimensional symmetry of the current algebra (2.7).

This group of transformations is characterised by a set of generators which satisfy, in addition,

(iii) $T^2 = \mathbf{1}$.

Then any T-duality transformation can be written as a product of a certain number of generators.

The analysis of the linear map T is very similar, at a formal level, to the one of the map R (see Section 3) which describes the boundary conditions: the first two

[4] In principle, one should demand that T-duality acts as an automorphism on the CFT of the holomorphic sector, but the automorphism group of the Virasoro algebra is trivial.

requirements will make the matrix T take exactly the same form as the matrix R. If we further impose that $T^2 = \mathbf{1}$ we obtain the two families of (non-trivial) solutions. The first one is given by

$$T_I = \begin{pmatrix} -1 & 0 & t_1 & 0 \\ 0 & -1 & t_2 & 0 \\ 0 & 0 & 1 & 0 \\ t_1 & t_2 & -\frac{1}{2}(t_1^2 + t_2^2) & 1 \end{pmatrix} .$$

It is a two-parameter family of T-duality transformations. If we define the order of T to be equal to the number of -1 eigenvalues then $\mathrm{ord}(T_I) = 2$. The second class of solutions is given by

$$T_{II} = \begin{pmatrix} \sin\theta & \cos\theta & t_1 & 0 \\ \cos\theta & -\sin\theta & t_2 & 0 \\ 0 & 0 & -1 & 0 \\ t_1 & t_2 & \frac{1}{2}(t_1^2 + t_2^2) & -1 \end{pmatrix} ,$$

and hence $\mathrm{ord}(T_{II}) = 3$.

In general, if we start with a boundary state characterised by the matrix R and perform a T-duality transformation given by the matrix T, the T-dual configuration will be described by the matrix

$$R' = R\,T .$$

Because both R and T are in the (double cover of the) adjoint group, so is R', and, moreover, given R' and R there exists a T which relates them, namely $R^{-1}R'$. Therefore, any two boundary states will be related by a T-duality. For example, the D-string we analysed before, which is defined by $R_I(\pi, r_1, r_2)$, can be described as the T-dual of the D3-brane defined by $R_I = \mathbf{1}$, where the T-duality transformation is given by $T = R_I(\pi, -r_1, -r_2)$. Similarly, a D0-brane configuration, say $R_{II}(\frac{\pi}{2}, 0, 0)$, can be obtained, via a transformation $T_{II} = R_{II}$, from the D3-brane configuration $R_I = \mathbf{1}$.

We now consider the two types of duality transformations separately.

5.1. T_I duality transformations

T_I maps between boundary states of the same type (described by matrices R belonging to the same class). Indeed, if we start with a boundary state characterised by the matrix $R_I(\phi, r_1, r_2)$ and apply the T-duality transformation characterised by $T_I(\theta, t_1, t_2)$ we obtain a (T-dual) boundary state described by $R_I(\phi', r_1', r_2')$ with

$$\begin{aligned} \phi' &= \phi + \theta \,, \\ r_1' &= r_1 + t_1 \cos\phi - t_2 \sin\phi \,, \\ r_2' &= r_2 + t_1 \sin\phi + t_2 \cos\phi \,. \end{aligned}$$

In particular, we recover that the D-string and the D3-brane without a background field are T-dual to each other.

On the other hand, if we start with a boundary state described by $R_{II}(\phi, r_1, r_2)$ and perform the transformation defined by $T_I(\theta, t_1, t_2)$, we obtain a configuration described by $R_{II}(\phi', r_1', r_2')$ with

$$\begin{aligned} \phi' &= \phi + \theta\,, \\ r_1' &= r_1 + t_1 \sin\phi + t_2 \cos\phi\,, \\ r_2' &= r_2 + t_1 \cos\phi - t_2 \sin\phi\,. \end{aligned}$$

Here we have to make a few remarks. We have seen in Section 4 that $R_{II}(\phi, r_1, r_2)$ is not diagonalisable in general, rather one has to restrict the parameters to satisfy (4.8). It turns out that generic $T_I(\theta, t_1, t_2)$ do not preserve this condition. If, on the other hand, we require that the dual $R_{II}(\phi', r_1', r_2')$ be diagonalisable as well, then we have to restrict ourselves to the transformations $T_I(\theta, t_1, t_2)$ such that ϕ', r_1' and r_2' satisfy a condition similar to (4.8). It turns out that the only $T_I(\theta, t_1, t_2)$ for which this equation is satisfied for all (ϕ, r_1, r_2) obeying (4.8) is the identity. Hence there is no subgroup of the T-duality subgroup which stabilises those boundary states with a D-brane interpretation.

5.2. T_{II} duality transformations

Again we start by noticing that T_{II} maps between boundary states of different type (described by R matrices belonging to different classes). Thus if we consider a boundary state given by $R_I(\phi, r_1, r_2)$ and apply a T-duality transformation characterised by the matrix $T_{II}(\theta, t_1, t_2)$ we obtain a T-dual boundary state described by $R_{II}(\phi', r_1', r_2')$ with

$$\begin{aligned} \phi' &= \theta - \phi \\ r_1' &= -r_1 + t_1 \cos\phi - t_2 \sin\phi \\ r_2' &= -r_2 + t_1 \sin\phi + t_2 \cos\phi\,. \end{aligned}$$

Notice that the dual R_{II} will not, in general, be diagonalisable. Thus if we consider a D-brane configuration R_I and perform a generic T_{II} transformation we will end up with a boundary state described by a nondiagonalisable R_{II}. In order for $R_{II}(\phi', r_1', r_2')$ to be diagonalisable $T_{II}(\theta, t_1, t_2)$ will have to satisfy an extra condition which follows from (4.8).

Finally, let us consider the effect of a $T_{II}(\theta, t_1, t_2)$ on a boundary state described by $R_{II}(\phi, r_1, r_2)$. The T-dual configuration is given by $R_I(\phi', r_1', r_2')$ with

$$\begin{aligned} \phi' &= \theta - \phi \\ r_1' &= -r_1 + t_1 \sin\phi + t_2 \cos\phi \\ r_2' &= -r_2 + t_1 \cos\phi - t_2 \sin\phi\,. \end{aligned}$$

6. D-brane configurations as classical solutions of Born-Infeld action in NW background

In Sections 3 and 4 we have discussed the requirements that a boundary state has to satisfy in order to preserve (some of) the conformal structure of the 2d theory and classified boundary states which can be interpreted as D-brane configurations. There is another approach to determining which D-branes can be embedded in a given curved background. Ignoring back reaction, one may start with the standard Born-Infeld action [27] for a Dp-brane probe moving in a curved space[5] and find if there are classical solutions describing static branes. The existence of a static solution (i.e. the absence of a static potential in the D-brane action) suggests that the corresponding brane is a BPS state.[6]

Since the only closed string fields that are non-trivial in the NW background are the metric and (NS-NS) antisymmetric tensor field, the relevant part of the D-brane action is (we ignore higher-derivative corrections)

$$I_p = \int d^{p+1}y \, \sqrt{-\det(\hat{G}_{mn} + \hat{B}_{mn} + F_{mn})} \, , \tag{6.1}$$

where x^μ are the coordinates of the $D = 10$ space and y^m are coordinates on the Dp-brane world-volume and $(m, n = 0, ..., p)$

$$\hat{G}_{mn} = G_{\mu\nu}\partial_m x^\mu \partial_n x^\nu, \quad \hat{B}_{mn} = B_{\mu\nu}\partial_m x^\mu \partial_n x^\nu, \quad F_{mn} = \partial_m A_n - \partial_n A_m.$$

It is straightforward to check that in the cases of $p = 3$, $p = 1$ and $p = 0$ the configurations $x^m = y^m$, $x^{p+1}, ..., x^9 = 0$, $F_{mn} = 0$ are, indeed, the solutions of the equations for x^μ and A_m which follow from (6.1).[7]

In the case of the D3-brane probe action in the static gauge ($x^m = y^m$) the equation for A_m is equivalent to the 'boundary' condition of conformal invariance of the corresponding string sigma model (see below).

[5]The NW background (times a torus) is an exact solution of both bosonic or type II string theory, so the discussion that follows applies to any of the two closed string theories.

[6]The BPS condition is usually phrased in terms of the residual *spacetime supersymmetry* and is thus translated (in the previously considered case of D-branes in the compactification manifold) into boundary conditions imposed on the spectral flow operator of the corresponding N=2 supersymmetric 2d theory [6, 9]. Residual supersymmetry condition [28] implies also satisfaction of the corresponding brane equations of motion. The 'no-force' condition is, in principle, more general than that of the residual supersymmetry, as it can be defined already in bosonic theory (the condition of the absence of the static potential was used, e.g., in [29] to determine possible composite BPS configurations of branes).

[7]The expansion of the action near the static brane configuration starts with term linear in velocity, reflecting the 'magnetic field' interpretation of the off-diagonal component of $G + B$ matrix.

JHEP06(1998)010

7. Relation to the sigma model approach

The boundary state approach can be applied to a wide range of string backgrounds [6, 8, 9, 10], which generically need not have a spacetime realisation in terms of sigma models. Indeed, all the necessary data is determined by the conformal structure of a particular background. Even in the cases in which an exact string background possesses a sigma model realisation, the boundary state configurations may not have a sigma model description. However, in order to interpret the boundary state solutions in terms of D-brane configurations in the target manifold one needs to understand the geometric content of these states. One should also keep in mind that the method of boundary states has its limitations, as it is not applicable to many string solutions described by conformal sigma models for which one does not know explicitly the exact generators of the (super)conformal algebra.

One of the remarkable features of the WZW models is that they fulfill both requirements (explicitly known conformal structure and sigma model realisation). However, since the boundary conditions in the boundary state approach are defined on the chiral currents rather than on the fields there is an obvious lack of geometric interpretation of the WZW boundary states, and in particular of the corresponding D-brane configurations [22, 10]. Moreover, the Ansatz for the relation between the currents at the boundary that one usually adopts is purely linear, while one might think that this may not be a natural assumption from the sigma model point of view, given that the background is curved.

In this section we will analyse this problem. We will start with a general WZW model (although some of the expressions below will hold for an arbitrary sigma model with 2-form field), and then we will consider the particular case of the Nappi-Witten background.

Let us start with an action of a generic WZW model on a 2-space with a disc topology with an additional interaction (1-form field A) at the boundary

$$S=\int_\Sigma \langle g^{-1}\partial g, g^{-1}\bar{\partial} g\rangle+\int_\Sigma g^*B+\int_{\partial\Sigma} g^*A\,. \tag{7.1}$$

Here the worldsheet Σ is a two-dimensional manifold with boundary $\partial\Sigma$ and B represents a particular choice for the antisymmetric tensor field (see [30])[8]. S may be viewed as a special case of an action for an open string propagating on a group manifold and coupled to A at the boundary.

The conditions of conformal invariance of this model in the bulk are satisfied as in the case of the WZW model on 2-sphere (we ignore the "back reaction" of the boundary coupling A on the bulk β-functions). The boundary conformal invariance condition [31], in general, may impose a constraint on A which should follow from the variation of the Born-Infeld action [31] $\int d^{10}x\sqrt{-\det(G+B+F)_{\mu\nu}}+O(\partial F,\partial B,R)$.

[8]We assume, for simplicity, that B is globally defined. This is true in the NW model, although it is not the case for compact Lie groups.

The leading-order condition of conformal invariance at the boundary can be represented as the Maxwell equation for $B+F$ on a curved space (group manifold) with metric G:

$$\partial_\mu \left[\sqrt{G} G^{\mu\nu} G^{\rho\sigma} (F+B)_{\nu\rho}\right] = 0\,. \tag{7.2}$$

In general, the solution for F depends on a specific choice of B. It is easy to check that this condition is indeed satisfied in the NW model for B as in (2.5) and $F_{\mu\nu} = 0$.[9]

If we vary the action S we get a bulk term which yields the same equations of motion as in the $\partial\Sigma = 0$ case, implying the conservation of the two sets of currents J_a and $\bar{J}_a$. We also get the boundary term

$$\int_{\partial\Sigma} d\tau (g^{-1}\delta g)^a \left[G_{ab}(g^{-1}\partial_\sigma g)^b - i(B_{ab}+F_{ab})(g^{-1}\partial_\tau g)^b\right]\Big|_{\sigma=0}^{\sigma=\pi}\,,$$

which yields a set of boundary conditions. The natural question is whether these conditions are related to the boundary conditions (3.1).

In order to address this question we introduce the coordinates x^μ on the group manifold (in the particular parametrisation (3) introduced in Section 2 these are a_1, a_2, u and v), and consider the left- and right-invariant vielbeins defined by

$$g^{-1}\partial_\mu g = e_\mu{}^a X_a\,, \qquad \partial_\mu g g^{-1} = \bar{e}_\mu{}^a X_a\,.$$

These vielbeins are related by $\bar{e}_\mu{}^a = e_\mu{}^b C_b{}^a$, where C denotes the adjoint action of the group, $gX_a g^{-1} = C_a{}^b X_b$.

The classical conserved currents are then

$$J_a = -G_{ab}\bar{e}_\mu{}^b \partial x^\mu\,, \qquad \bar{J}_a = G_{ab} e_\mu{}^b \bar{\partial} x^\mu\,. \tag{7.3}$$

The surface term in the infinitesimal variation of the action can be written as

$$\int_{\partial\Sigma} d\tau \delta x^\mu p_\mu \Big|_{\sigma=0}^{\sigma=\pi}$$

where p_μ (which is the component of the 2-momentum normal to the boundary $\partial\Sigma$) is given by

$$p_\mu = G_{\mu\nu}\partial_\sigma x^\nu - i(B_{\mu\nu}+F_{\mu\nu})\partial_\tau x^\nu\,,$$

where $G_{\mu\nu} = e_\mu{}^a G_{ab} e_\nu{}^b$, $B_{\mu\nu} = e_\mu{}^a B_{ab} e_\nu{}^b$ and $F_{\mu\nu} = e_\mu{}^a F_{ab} e_\nu{}^b$. Thus having Neumann boundary conditions in all directions means imposing $p_\mu|_{\partial\Sigma} = 0$ for all μ. Using (7.3), we can write p_μ in terms of the holomorphic and antiholomorphic currents as follows:

$$p_\mu = -[\delta_\mu{}^\rho - (B+F)_{\mu\nu}G^{\nu\rho}]\bar{e}_\rho{}^a J_a - [\delta_\mu{}^\rho + (B+F)_{\mu\nu}G^{\nu\rho}]e_\rho{}^a \bar{J}_a\,.$$

[9]It seems likely that for a "natural" choice of B this condition is always satisfied for $F=0$ in a generic WZW model. This is easy to check directly for some simple cases like $SU(2)$ and $SL(2,R)$ WZW models.

JHEP06(1998)010

Then the Neumann boundary conditions take the following compact form

$$J + \mathbf{M}\bar{J} = 0\,, \tag{7.4}$$

where

$$\mathbf{M} \equiv \bar{e}^{-1}\frac{\mathbf{1} + (B+F)G^{-1}}{\mathbf{1} - (B+F)G^{-1}}e\,. \tag{7.5}$$

This is a generalisation of the familiar expression for the boundary conditions for an open string in a constant (G, B, F) background [1]. Note that the matrix $\mathbf{M}$ is no longer constant but field-dependent. Moreover, the boundary conditions for the Lie algebra valued currents depend explicitly on the vielbeins.

The field dependent nature of the above matrix allows it to acquire, for special values of the fields, the -1 eigenvalues which are interpreted as Dirichlet boundary conditions. In the constant background (flat) case such a transition from Neumann to Dirichlet boundary conditions is impossible to attain for any *finite* values of the field $(B + F)G^{-1}$.

Let us now describe the sigma model realisation of a Dp-brane, with $p + 1$ strictly smaller than the dimension of the target manifold (in our case $p < 3$). In this case some of the fields satisfy Dirichlet boundary conditions, and the way we implement this is by imposing [27] that x^μ at the boundary are given by a set of functions $\{f^\mu\}$ defined on the worldvolume of the Dp-brane

$$x^\mu|_{\partial\Sigma} = f^\mu(y^m)\,, \tag{7.6}$$

where y^m denote the coordinates on the $(p + 1)$-dimensional submanifold determined by the worldvolume of the Dp-brane. The infinitesimal variation of the spacetime fields at the boundary is given by $\delta x^\mu|_{\partial\Sigma} = \partial_m f^\mu \delta y^m$, so that the boundary term in the variation of the WZW action becomes

$$\int_{\partial\Sigma} d\tau \delta y^m p_m \Big|_{\sigma=0}^{\sigma=\pi}\,,$$

$$p_m = \partial_m f^\mu G_{\mu\nu}\partial_\sigma x^\nu - i(B+F)_{mn}\partial_\tau y^n\,,$$

where $B_{mn} = \partial_m f^\mu B_{\mu\nu}\partial_n f^\nu$ and $F_{mn} = \partial_m f^\mu F_{\mu\nu}\partial_n f^\nu$ are the antisymmetric tensor and the background gauge field induced at the worldvolume of the Dp-brane. In this case the $p + 1$ Neumann boundary conditions become $p_m|_{\partial\Sigma} = 0$ or, in terms of the currents

$$\partial_m f^\mu[\mathbf{1} - (B+F)G^{-1}]_\mu{}^\nu \bar{e}_\nu{}^a J_a + \partial_m f^\mu[\mathbf{1} + (B+F)G^{-1}]_\mu{}^\nu e_\nu{}^a \bar{J}_a = 0\,. \tag{7.7}$$

7.1. The D3-brane

Let us compute the matrix of boundary conditions in (7.4) in the particular case of the Nappi-Witten background. The vielbeins e and $\bar{e}$ are given by

$$e = \begin{pmatrix} \cos u & -\sin u & 0 & \frac{1}{2}a_2 \\ \sin u & \cos u & 0 & -\frac{1}{2}a_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \bar{e} = \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{2}a_2 \\ 0 & 1 & 0 & \frac{1}{2}a_1 \\ a_2 & -a_1 & 1 & -\frac{1}{2}(a_1^2 + a_2^2) \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

If we assume that $F = 0$ and we take the antisymmetric tensor field to be as in (2.5) (this choice is consistent with conformal invariance) we find for the matrix of the boundary conditions

$$\mathbf{M} = \begin{pmatrix} \cos u & -\sin u & 0 & 0 \\ \sin u & \cos u & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{7.8}$$

Notice that this matrix is of the form (3.4), with the parameter ϕ given by the field u and $r_1, r_2 = 0$. Different choices for B (while keeping $F = 0$) will yield different $\mathbf{M}$ (different boundary conditions), but they will still be described by a matrix of the type $R_I(\phi, r_1, r_2)$ with the parameters given by certain functions of the fields a_i, u, v.

An unusual feature of the boundary conditions described by (7.8) is their oscillatory character. In particular, when $u = (2n+1)\pi$, with n=integer, we have two Dirichlet directions, corresponding to a_1 and a_2.

It is instructive to rederive these results in a different parametrisation of the group G

$$g = e^{x_1 P_1} e^{uJ} e^{x_2 P_1 + vK},$$

in which $\{x^\mu\} = \{x_1, x_2, u, v\}$ and the spacetime metric reads

$$ds^2 = dx_i dx_i + 2\cos u \; dx_1 dx_2 + b du^2 + 2 du dv \,.$$

This coordinate system is singular for $u = \pi m$, m=integer. If we choose the antisymmetric tensor to be $B = \frac{1}{2}\epsilon_{ij} x_i dx_j du$ and compute $\mathbf{M}$ we get exactly the same matrix as in (7.8). Here the oscillatory character of the boundary conditions is very similar to the one of the metric itself. In particular, the Dirichlet conditions appear at values of u which correspond to the singularities of the coordinate system (and the metric).

7.2. The D1-brane

We now consider the sigma model description of the D-string found in Section 4. The two Dirichlet boundary conditions are described as in (7.6), with the functions f^μ given by (4.6). Here we are in the static gauge since the two-dimensional worldsheet of the D-string coincides with the surface defined by the coordinates u and v.

In this case, if we take $F = 0$ and B given by (2.5), the Neumann boundary conditions (7.7) become

$$J_3 + \bar{J}_3 = 0\,, \qquad J_4 + \bar{J}_4 = 0\,,$$

which agree with the Neumann boundary conditions obtained from R_I.

It may seem that we have unnecessarily restricted ourselves to a particular D-string configuration, namely, the one given by (4.6), instead of considering the most general one in (4.1)-(4.4). One can easily derive the Neumann boundary conditions in the most general case: one obtains a set of two relations which agree with the Neumann boundary conditions coming from the boundary state approach *only* for $r_1, r_2 = 0$. This result is not surprising since the equations describing the D-string have been obtained under the assumption that r_1 and r_2 are constant parameters. On the other hand, we have seen in the case of the D3-brane that the two parameters, r_1 and r_2, are generically field-dependent. In order to write down a sigma-model describing a more general D1-brane configuration one should probably consider functions f^μ which are field-dependent.

7.3. The D0-brane

Finally, let us describe a sigma model realisation of the D0-brane configuration obtained in Section 4 and characterised by R_{II}. In this case the fields x^μ satisfy boundary conditions of the form (7.6), where the functions f^μ are given by (4.9)-(4.12) (we consider here only the case $\phi \neq \frac{3\pi}{2}$, the case $\phi = \frac{3\pi}{2}$ being very similar). If we compute the Neumann boundary condition (2.5), with F and B as in the previous case, we obtain

$$\begin{aligned}(1+\sin\phi)(J_1 + \bar{J}_1) + \cos\phi(J_2 + \bar{J}_2) &+ \\ + \tfrac{1}{2}\left[r_1(1+\sin\phi) + r_2\cos\phi\right](J_4 + \bar{J}_4) &= 0\,,\end{aligned}$$

which is nothing but the Neumann condition corresponding to R_{II}.

Acknowledgments

We are grateful to C. Klimčik and Y. Lozano for valuable discussions, and to J.M. Figueroa-O'Farrill for helpful discussions and for drawing the pictures in the Appendix. This work was supported in part by PPARC, the European Commission TMR programme grant ERBFMRX-CT96-0045 and the NSF grant PHY94-07194.

A. Explicit space-time embedding of the D-string solution

Here we describe the D-string configuration in more detail. It is convenient to change variables in the Nappi–Witten spacetime to $x^\mu = (x^0, x^1, x^2, x^3)$ defined in terms of (a_i)

as follows:

$$\begin{aligned}
x^0 &= \frac{1}{\sqrt{2}}\left[a_4 - (1+\tfrac{1}{2}b)a_3 + \tfrac{1}{4}(r_1 a_1 + r_2 a_2)\right]\\
x^1 &= a_1\\
x^2 &= a_2\\
x^3 &= \frac{1}{\sqrt{2}}\left[a_4 + (1-\tfrac{1}{2}b)a_3\right] .
\end{aligned}$$

The Nappi–Witten metric in these coordinates becomes

$$\begin{aligned}
ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu - \tfrac{1}{32}(r_1 dx^1 + r_2 dx^2)^2 + \tfrac{1}{2\sqrt{2}}(r_1 dx^1 + r_2 dx^2)dx^0 + \\
+ \tfrac{1}{\sqrt{2}}\left(x^1 dx^2 - x^2 dx^1\right)\left(dx^3 - dx^0 + \tfrac{1}{4\sqrt{2}}(r_1 dx^1 + r_2 dx^2)\right) .
\end{aligned}$$

The virtue of these coordinates is that the intrinsic D-string time and the time of the ambient spacetime agree. Indeed, the explicit embedding of D-string worldsheet is given by

$$\begin{aligned}
x^0(\sigma,\tau) &= \tau\\
x^1(\sigma,\tau) &= \tfrac{1}{2}r_1 \sin\tfrac{1}{\sqrt{2}}(\sigma-\tau) + \tfrac{1}{2}r_2\left(\cos\tfrac{1}{\sqrt{2}}(\sigma-\tau) - 1\right)\\
x^2(\sigma,\tau) &= -\tfrac{1}{2}r_1\left(\cos\tfrac{1}{\sqrt{2}}(\sigma-\tau) - 1\right) + \tfrac{1}{2}r_2 \sin\tfrac{1}{\sqrt{2}}(\sigma-\tau)\\
x^3(\sigma,\tau) &= \sigma - \tfrac{1}{8\sqrt{2}}\varrho^2 \sin\tfrac{1}{\sqrt{2}}(\sigma-\tau) ,
\end{aligned}$$

where $\varrho \equiv \sqrt{r_1^2 + r_2^2}$ and

$$\begin{aligned}
\sigma &= \frac{1}{\sqrt{2}}\left(\left(1+\tfrac{1}{2}b+\tfrac{1}{8}\varrho^2\right)\alpha + \beta\right)\\
\tau &= \frac{1}{\sqrt{2}}\left(\left(1-\tfrac{1}{2}b-\tfrac{1}{8}\varrho^2\right)\alpha - \beta\right) ,
\end{aligned}$$

relative to which the metric (4.5) becomes simply $ds^2 = d\sigma^2 - d\tau^2$.

Therefore the $x^0 =$ constant hyperplane cuts the D-string worldsheet in the D-string itself. The snapshots in fig. 1 illustrate the D-string embedded in the three-dimensional hypersurface $x^0 = 0$, for different values of ϱ.

B. N=2 supersymmetric generalisation

The above discussion of the bosonic NW model admits an N=2 generalisation. Indeed, the N=1 extension of (2.1) describes a superconformal theory with the superconformal algebra generated by (T,G). Further, if we consider the algebra (2.2) and we define $t_+ = \{P^1_+, P^2_+\}$ and $t_- = \{P^1_-, P^2_-\}$, where

$$P^1_\pm = \tfrac{1}{2}(P_1 \mp iP_2) , \qquad P^2_\pm = \tfrac{1}{2}(J \mp iK) , \tag{B.1}$$

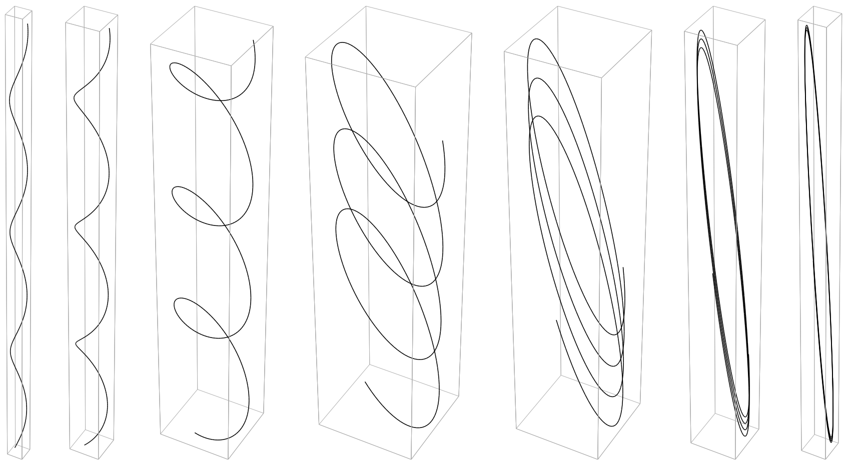

Figure 1: $x^0 = \tau = 0$ snapshots for $\varrho = 1, 2, 5, 10, 20, 50, 100$.

we can easily check that $[t_\pm, t_\pm] \subset t_\pm$, which is the condition for the N=1 theory to admit an N=2 extension, with the superconformal algebra generated by $(\mathsf{T}, \mathsf{G}^\pm, \mathsf{J})$. In this case we can apply the boundary state approach as described in [9] and determine which of the solutions that we have found for the bosonic boundary states (R_I, R_{II}) survive as boundary states of the N=2 model. The boundary conditions in the N=2 case fall into the two types, A and B, and the additional requirement that the boundary state described by R has to satisfy can be stated as follows:

(i) A–type boundary conditions

$$R\ A = -A\ R\ ; \tag{B.2}$$

(ii) B–type boundary conditions

$$R\ A = A\ R, \tag{B.3}$$

where A is the complex structure on $\mathfrak{g}$, which in the NW case can be taken to be

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\ . \tag{B.4}$$

We can now check explicitly the existence of each of the solutions (R_I, R_{II}) found in Section 3. We find that R_{II} yields no solutions for either of the two types of boundary conditions, whereas R_I only yields solutions for the B-type boundary conditions. These are given by (3.4) with $r_1, r_2 = 0$, i.e. are of the form $R_I(\phi, 0, 0)$.

References

[1] C. Callan, C. Lovelace, C. Nappi and S. Yost, *Loop corrections to superstring equations of motion*, *Nucl. Phys.* **B 308** (1988) 221.

[2] J. Polchinski and Y. Cai, *Consistency of open superstring theories*, *Nucl. Phys.* **B 296** (1988) 91.

[3] M. Li, *Boundary states of D-branes and dy-strings*, *Nucl. Phys.* **B 460** (1996) 351 [hep-th/9510161].

[4] C. Callan, Jr. and I. Klebanov, *D-brane boundary state dynamics*, *Nucl. Phys.* **B 465** (1996) 473 [hep-th/9511173].

[5] J. Polchinski, *TASI lectures on D-branes*, in *Proceedings of the TASI School 1996*, 1996. [hep-th/9611050].

[6] H. Ooguri, Y. Oz and Z. Yin, *D-branes on Calabi–Yau spaces and their mirrors*, *Nucl. Phys.* **B 477** (1996) 407 [hep-th/9606112].

[7] K. Becker, M. Becker, D.R. Morrison, H. Ooguri, Y. Oz and Z. Yin, *Supersymmetric cycles in exceptional holonomy manifolds and Calabi–Yau 4-folds*, *Nucl. Phys.* **B 480** (1996) 225 [hep-th/9608116].

[8] M. Kato and T. Okada, *D-branes on group manifolds*, *Nucl. Phys.* **B 499** (1997) 583 [hep-th/9612148].

[9] S. Stanciu, *D-branes in Kazama–Suzuki models*, hep-th/9708166.

[10] A. Recknagel and V. Schomerus, *D-branes in Gepner models*, hep-th/9712186.

[11] J. Fuchs and C. Schweigert, *Branes: from free fields to general backgrounds*, hep-th/9712257.

[12] C. Nappi and E. Witten, *A WZW model based on a non-semi-simple group*, *Phys. Rev. Lett.* **71** (1993) 3751 [hep-th/9310112].

[13] A. Tseytlin, *Exact solutions of closed string theory*, *Class. and Quant. Grav.* **12** (1995) 2365 [hep-th/9505052].

[14] E. Witten, *Nonabelian bosonization in two dimensions*, *Comm. Math. Phys.* **92** (1984) 455.

[15] D. Olive, E. Rabinovici and A. Schwimmer, *A class of string backgrounds as a semiclassical limit of WZW models*, *Phys. Lett.* **B 321** (1994) 361, [hep-th/9311081].

[16] E. Kiritsis and C. Kounnas, *String propagation in gravitational backgrounds*, *Phys. Lett.* **B 320** (1994) 264; Addendum: *ibid.* **325** (1994) 536 [hep-th/9310202].

[17] E. Kiritsis, C. Kounnas and D.Lüst, *Supersymmetric gravitational wave backgrounds with space-time supersymmetry*, *Phys. Lett.* **B 331** (1994) 321 [hep-th/9404114].

JHEP06(1998)010

[18] N. Mohammedi, *On bosonic and supersymmetric current algebras for nonsemisimple groups*, *Phys. Lett.* **B 325** (1994) 371 [hep-th/9312182].

[19] J.M. Figueroa-O'Farrill and S. Stanciu, *Nonsemisimple Sugawara constructions*, *Phys. Lett.* **B 327** (1994) 40 [hep-th/9402035].

[20] J.M. Figueroa-O'Farrill and S. Stanciu, *N=1 and N=2 cosets from gauged supersymmetric WZW models*, *Mod. Phys. Lett.* **A 12** (1997) 1677 [hep-th/9511229].

[21] J. Figueroa-O'Farrill and S. Stanciu, *Nonreductive WZW models and their CFTs*, *Nucl. Phys.* **B 458** (1996) 137 [hep-th/9506151].

[22] N. Ishibashi, *The boundary and crosscap states in conformal field theories*, *Mod. Phys. Lett.* **A 4** (1989) 251.

[23] M. Berkooz, M. Douglas and R. Leigh, *Branes intersecting at angles*, *Nucl. Phys.* **B 480** (1996) 265 [hep-th/9606139].

[24] E. Kiritsis, *Exact duality symmetries in CFT and string theory*, *Nucl. Phys.* **B 405** (1993) 109 [hep-th/9302033].

[25] E. Alvarez, L. Alvarez-Gaumé and Y. Lozano, *A canonical approach to duality transformations*, *Phys. Lett.* **B 336** (1994) 183 [hep-th/9406206].

[26] A. Giveon and E. Witten, *Mirror symmetry as a gauge symmetry*, *Phys. Lett.* **B 332** (1994) 44 [hep-th/9404184].

[27] R. Leigh, *Dirac-Born-Infeld action from Dirichlet sigma-model*, *Mod. Phys. Lett.* **A 4** (1989) 2767.

[28] K. Becker, M. Becker and A. Strominger, *Fivebranes, membranes and non-perturbative string theory*, *Nucl. Phys.* **B 456** (1995) 130 [hep-th/9507158].

[29] A. Tseytlin, *'No-force' condition and BPS combinations of p-branes in 11 and 10 dimensions*, *Nucl. Phys.* **B 487** (1997) 141 [hep-th/9609212].

[30] C. Klimčik and P. Severa, *Open strings and D-branes in WZNW models*, *Nucl. Phys.* **B 488** (1997) 653 [hep-th/9609112].

[31] A. Abouelsaood, C. Callan, C. Nappi and S. Yost, *Open strings in background gauge fields*, *Nucl. Phys.* **B 280** (1987) 599.

JHEP06(1998)010